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Landscape approach for pinned elastic interfaces

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Abstract

We discuss the large scale effective free energy landscape for elastic objects pinned by a random potential. In the static approach, converging analytical results show that this landscape consists in a succession of parabolic wells of random depth, matching on singular points where the effective force is discontinuous. We discuss the consequences for the dynamics of these pinned interfaces.

Keywords: Elastic manifolds; Randomness; Renormalisation

1. Introduction

The physics of elastic objects pinned by random impurities is of fundamental importance both from a theoretical point of view (many of the specific difficulties common to disordered systems are at stake) and for applications: the pinning of flux lines in superconductors [1–3], of dislocations, of domain walls in magnets, or of charge density waves [4,5], controls in a crucial way the properties of these materials. Interestingly, this problem is also intimately connected to surface growth [6], to fracture [7] and to turbulence [8].

One may first consider the equilibrium problem, where the probability distribution of the position of such a manifold is assumed to be given by Boltzmann law (the dynamical problem, where one discusses the relaxation towards this equilibrium – possibly never reaching it – will be discussed below). Two different general approaches have been proposed to describe this *static* problem, for which perturbation theory fails. The first one is the variational replica method which combines a Gaussian trial Hamiltonian with 'replica symmetry breaking' (RSB) to obtain a quantitative description of the low temperature, strongly pinned phase [9–11]. The second is the 'functional renormalisation group' (FRG) which aims at constructing the correlation function for the effective pinning potential acting on long wavelengths using renormalisation group (RG) ideas [12,13]. These two methods seemed somewhat difficult to compare directly since the quantities one computes in each of them are quite different. Recently we

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obtained some new results on the connections between these two approaches [14]. Besides its mathematical interest, this work also sheds some new light on the physical mechanisms, and particularly the 'landscape aspects' underlying the glassy behaviour of the pinned phase. We show that both formalisms are indeed struggling to describe an awkward reality: the effective, long wavelength pinning potential is a succession of parabolic wells of random depth, matching on singular points where the effective force (i.e. the derivative of the potential) is discontinuous. These discontinuities induce a singularity in the effective potential correlation function, and are encoded in the replica language by the RSB. The replica calculation furthermore provides an explicit construction of this effective (random) potential. This allows us to get more information, for instance on the depth of the potential minima, and also to make more explicit the assumptions on which the FRG approach relies.

We consider the general problem of pinned elastic manifolds described by the Hamiltonian:

$$\mathcal{H}(\{\phi(\mathbf{x})\}) = \int \mathrm{d}^D\mathbf{x} \left[\frac{c}{2} \left(\frac{\partial \phi(\mathbf{x})}{\partial \mathbf{x}} \right)^2 + V_0(\mathbf{x}, \phi(\mathbf{x})) \right], \tag{1}$$

where $\mathbf{x}$ is a D-dimensional vector labelling the internal coordinates of the object, and $\phi(\mathbf{x})$ an N-dimensional vector giving the position in physical space of the point labelled $\mathbf{x}$. Various values of D and N actually correspond to interesting physical situations. For example, $D = 3$, $N = 2$ describes the elastic deformation of a vortex lattice (after a suitable anisotropic generalisation of Eq. (1)), $D = 2$, $N = 1$ describes the problem of domain walls pinned by impurities in three-dimensional space, while $D = 1$ corresponds to the well-known directed polymer (or single flux line) in an $N + 1$-dimensional space. The elasticity of the structure is characterised by the modulus c. Obviously for each specific case the elastic term should be adapted (for instance in vortex lattices one should introduce the three moduli C_{11}, C_{44}, C_{66}), but in this presentation we want to stay at a general level and we shall discuss the simplest case, leaving aside such (important) details. The pinning potential $V_0(\mathbf{x}, \phi(\mathbf{x}))$ is a random function, which we shall choose to be Gaussian distributed with a correlation function:

$$\overline{V_0(\mathbf{x}, \phi) V_0(\mathbf{x}', \phi')}_c = NW\delta^D(\mathbf{x} - \mathbf{x}') R_0 \left(\frac{(\phi - \phi')^2}{N} \right), \tag{2}$$

where W measures the strength of the pinning potential. We shall here concentrate on the case where the correlation function is short ranged (although the long range case is also interesting), and we shall choose for convenience $R_0(y) = \exp(-y/2\Delta^2)$, where Δ is the correlation length of the random potential.

2. Statics

A lot of efforts have been devoted to this type of problem, particularly for computing the wandering exponent characteristic of the transverse fluctuations of the manifold at low temperature, which is known to be non trivial below four dimensions. Simple scaling arguments, which are rather successful [15], are complemented by microscopic computations using the two methods mentioned before [9,13], leading to some approximate values of the exponents. We shall not discuss this aspect here, but we want to focus on the physical pinning mechanisms. One aim of the theory is to understand how the microscopic pinning potential will affect the elastic manifold on long length scales, relevant for macroscopic measurements. In other words, one would like to construct the *effective* pinning potential seen by a low wavevector mode of the structure, after thermalising the modes with shorter length scales. Both the FRG and the replica approach propose an approximate construction of this effective potential.

The FRG method consists in writing down a recursion relation for the correlation function of the potential acting on 'slow' modes $\phi_<$, after 'fast' modes $\phi_>$ (corresponding to wavevectors in the high-momentum shell $[\Lambda/b, \Lambda]$) have been integrated out using perturbation theory, and after a proper coarse graining of the variables both in x space and in ϕ space [13]. Assuming that the renormalised random pinning potential still has a Gaussian distribution, one can write a recursion relation for the two-point correlation function of the potential. Close to four dimensions, to first order in $\epsilon = 4 - D$, one can argue that this correlation function keeps the form (2), where the function $R_0(y)$ is replaced by a scale dependent correlation $R_l(y)$. The iteration of the recursion equation from the 'initial' condition $R(y) = R_0(y)$ converges towards the *fixed point* $R^*(y)$, describing the long wavelength properties, which has the singular small y expansion [13]

$$R^*(y) - R^*(0) = \epsilon y[a_1 - a_{3/2}\sqrt{y}] + \cdots . \tag{3}$$

In terms of the effective *force* f acting on the manifold (defined as minus the derivative of the effective potential with respect to ϕ), one finds that the force correlation function behaves as

$$\overline{[f^*(\phi) - f^*(\phi')]^2} = 12\epsilon a_{3/2}|\phi - \phi'|. \tag{4}$$

The two main assumptions underlying this computation are the use of perturbation theory in the decimation procedure on the one hand, and the assumption of Gaussian statistics on the other hand. Within this framework the effective force acting on the manifold would seem to behave, for $N = 1$, as a *random walk* in ϕ space.

The replica approach is, in some sense, more ambitious, since it provides an explicit probabilistic construction of the effective disordered potential seen by the manifold. On the other hand, it is based on a non-perturbative variational method (of the Hartree type) which is exact only when $N \to \infty$. Furthermore it turns out that the probabilistic construction of the effective potential is somewhat intricate [8,9], so that the computation of the effective large scale correlation of this potential is not easy. This is basically the computation which was done in [14], and for which we describe here only the general idea. One first isolates a particular, very slow mode $\mathbf{k}_0 \to 0$. The effective force acting on $\varphi_0 \equiv \varphi(\mathbf{k}_0)$ is $f^\mu_\Omega(\varphi_0) = -(1/\beta)(\partial/\partial\varphi^\mu_0)\ln\mathcal{P}_\Omega(\varphi_0)$, where $\mathcal{P}_\Omega(\varphi_0)$ is the probability to observe φ_0 for a given realisation of the random pinning potential Ω (notice that all the other modes are supposed to be thermalised). One computes the correlation function of f with the aid of replicas through the object:

$$\overline{f^\mu_\Omega(\varphi_0) f^\nu_\Omega(\varphi'_0)} = \lim_{n\to 0}\frac{4}{n^2}\frac{\partial^2}{\partial\varphi^\mu_0\,\partial\varphi'^\nu_0}\overline{[\mathcal{P}_\Omega(\varphi_0)]^{n/2}[\mathcal{P}_\Omega(\varphi'_0)]^{n/2}}, \tag{5}$$

which is directly calculable within the Gaussian RSB Ansatz. A somewhat elaborate computation leads, in the limits $k_0 \to 0$, $D \to 4$, $N \to \infty$, to a two-point correlation for the large scale effective pinning potential which has the same structure as the one derived from the FRG, confirming the convergence of the results of the two methods in this regime. This was hoped for, since the FRG is supposed to hold when $\epsilon \to 0$ and the Gaussian variational replica approach is supposed to hold when $N \to \infty$.

Things become more tricky, and more interesting, when one considers the physical interpretation of the replica computation, which also allows to compute higher moments of the large scale potential (V^*_Ω) statistics. The Gaussian variational ansatz with RSB does not mean at all that the statistics of V^*_Ω is Gaussian. Let us first discuss the replica construction of the effective potential in the case $D = 1$ where a simpler, one step RSB, solution holds [8,9]. In this case, one has

$$V^*_\Omega(\varphi) = -\frac{1}{\beta}\ln\left[\sum_\alpha \exp\left(-\beta F_\alpha - \frac{(\varphi-\varphi_\alpha)^2}{u_c\Delta^2}\right)\right], \tag{6}$$

where α label the 'states', centred around φ_α and of free-energy F_α, both depending on the 'sample' Ω. The priviledged positions φ_α are uniformly distributed, while the free energies F_α are exponentially distributed:

$$\rho(F_\alpha) \propto_{F_\alpha\to-\infty} \exp(-\beta u_c|F_\alpha|). \tag{7}$$

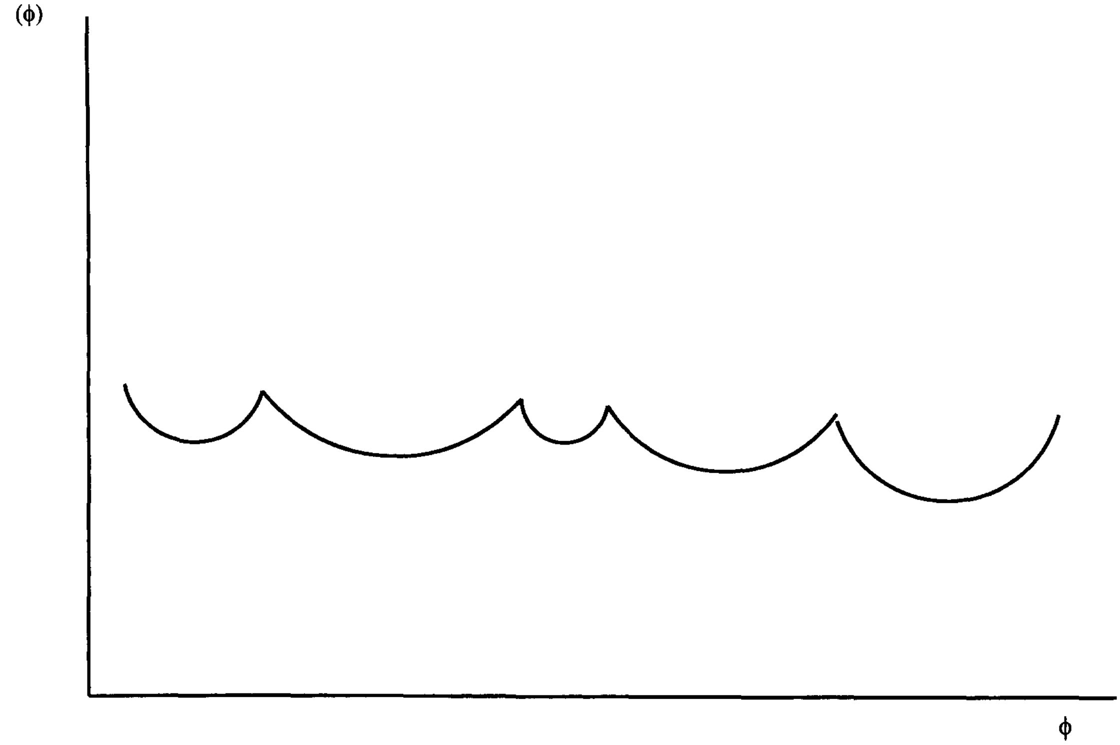

Fig. 1. Schematic view of the effective energy landscape as a succession of parabolic wells matching at singular point. This picture actually corresponds to a 'one-step' replica symmetry breaking scheme.

The full distribution of the effective force $\partial V_{\Omega}^{*}/\partial\varphi_0$ (corresponding to the velocity in the Burgers turbulence problem) was analysed in detail in [8], in the turbulence language. Translated into the present language, one finds that the potential has for $N = 1$ the shape drawn in Fig. 1: it is made of parabolas matching at angular points. The singular behaviour of the force–force correlation function, Eq. (4), is due to the fact that with a probability proportional to the 'distance' $|\varphi_0 - \varphi_0'|$, there is a shock which gives a *finite* contribution to $f(\varphi_0) - f(\varphi_0')$. This means in particular that all the moments $\overline{|f(\varphi_0) - f(\varphi_0')|^p}$ grow as $|\varphi_0 - \varphi_0'|$ for $p \geq 1$, instead of $|\varphi_0 - \varphi_0'|^{p/2}$ as for Gaussian statistics. In the case of continuous RSB, the construction of the effective potential is more complicated. Basically it is recursively constructed via a set of 'Matrioshka doll' Gaussians [9,14]. It is schematically drawn in Fig. 2 for the transverse fluctuations $\phi(l) - \phi(0)$. The singular structure of the two-point correlations $R(y)$ for small y still holds.

The relation with Burgers' equation is actually quite interesting. Keeping $N = 1$ for simplicity, consider a toy model for the FRG mode elimination in which the renormalized effective potential is defined as

$$\beta V_R(\varphi_<) = -\ln\left[\int \mathrm{d}\varphi_> \mathrm{e}^{-\beta[(cA^2/2)\varphi_>^2 + V_0(\varphi_< + \varphi_>)]}\right]. \tag{8}$$

This means that $V_R(\varphi_<)$ is precisely the Cole–Hopf solution of the Burgers equation [16]:

$$\frac{\partial V(\varphi,t)}{\partial t} = \frac{1}{2\beta cA^2}\frac{\partial^2 V(\varphi,t)}{\partial\varphi^2} - \frac{cA^2}{2}\left(\frac{\partial V(\varphi,t)}{\partial\varphi}\right)^2 \tag{9}$$

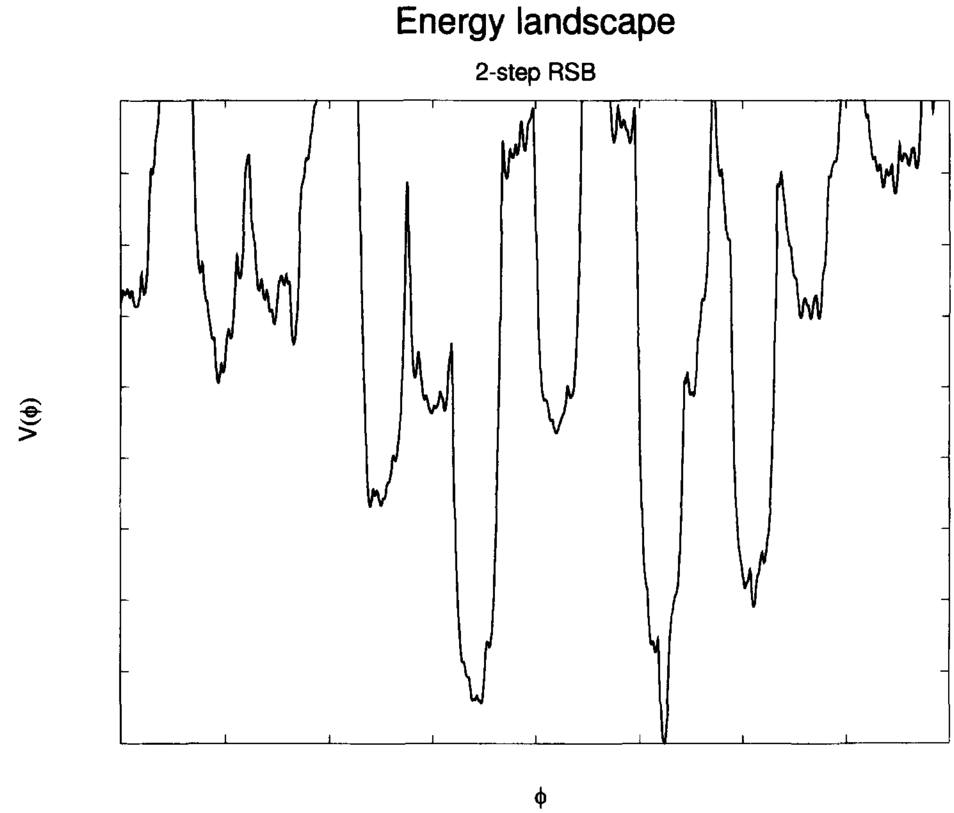

Fig. 2. Multiscale energy landscape corresponding to a full replica symmetry breaking scheme. In this case, the construction is that of parabolas within parabolas, in a hierarchical manner. The depth of the wells (and thus also the height of the barriers) typically grows as $|\phi - \phi'|^{\theta/\zeta}$. The figure actually corresponds to a two-step breaking scheme, with $u_1 = 0.5$ and $u_0 = 0.05$. The inset is a zoom on a particular region, showing the first level of Gaussians.

with

$$V(\varphi, t = 0) = V_0(\varphi), \qquad V_R(\varphi) = V(\varphi, t = 1). \tag{10}$$

As is well-known [16,17], a random set of initial conditions (here the bare pinning potential acting on φ) develops shocks which separates as time grows, between which the 'potential' $V(\varphi)$ has a parabolic shape. Elimination of fast modes in a disordered system thus naturally generates a 'scalloped' potential, with singular points separating potential wells – the famous 'states' appearing in the replica theory.

The reason why the FRG succeeds in getting correctly the singularity in $R(y)$ while it uses the wrong assumption of the large scale potential being Gaussian is not totally clear. To get some idea about this issue, one must keep in mind that the effective potential calculated within the FRG procedure involves an extra step which we have not performed within the replica construction, which is a coarse graining of the ϕ variables. In the FRG calculation, one restricts to configurations which are such that ϕ is constant on scales l, and scales as l^{ζ}. The correct choice of ζ then ensures that there are only a few shocks on the scale l. This is perhaps why the FRG can still be controlled, the departure from Gaussian statistics being in some sense (which we do not fully understand) 'weak'.

3. Dynamics

We believe that our construction of the effective pinning potential could turn out to be very useful in order to understand the *dynamics* of such objects at finite N. It suggests for instance to analyse the relaxation of one mode in terms of hops between the different minima ('traps'), corresponding to metastable long wavelength configurations. Of course one must be cautious about this approach because it is not clear that one can really analyse the relaxation of one mode in its effective static potential: this amounts to assume that all the other modes are already equilibrated, which is not guaranteed since they are themselves slow! Keeping this in mind, it is nevertheless interesting to notice that the trapping time distribution, which is controlled by the distribution of F_α, is a broad distribution without a first moment. More precisely, the lifetime of each 'trap' is activated $\tau \simeq \tau_0 \exp(\beta \Delta E)$, which is distributed – in the full RSB picture – as a power-law $\tau^{-1-u(k)}$ for large τ, where the exponent $u(k) \propto k^\theta$ depends on the 'size' of the jump (i.e. the mode involved in the change of conformation), small $u(k)$ corresponding to large wavelengths. (θ is the so-called energy exponent which is close to 2 in dimension $D \sim 4$). Then, as emphasised in [18] where precisely the same 'trap' picture was advocated for spin-glasses, the dynamics becomes non-stationary and aging effects appear at low temperatures and/or long-wavelengths such that $u(k) < 1$.

A direct analytic study of the dynamics has also been developed by mean field methods which hold for large N. Using the dynamical field theory approach [19–21], one can write [22] in this limit (it is important that the large N limit is taken before the large time limit in this approach) some coupled equations for the correlation:

$$C(x,t;x',t') = \left\langle \frac{1}{N} \sum_\alpha \phi_\alpha(x,t)\phi_\alpha(x',t') \right\rangle \tag{11}$$

and the response:

$$r(x,t;x',t') = \left\langle \frac{1}{N} \sum_\alpha \frac{\partial \phi_\alpha(x,t)}{\partial \eta_\alpha(x',t')} \right\rangle. \tag{12}$$

These dynamical equations involve a memory kernel and an effective noise which are determined self-consistently. The understanding of this mean field dynamics parallels that of spin-glasses. For simplicity we shall discuss the case of zero dimension. At high temperature one finds a solution which is, at large enough time, both time translational invariant (TTI) and obeys the fluctuation dissipation theorem (FDT). At a critical temperature the relaxation time becomes infinite [22]. In the low temperature phase, as first recognised in spin-glasses by Cugliandolo and Kurchan [23], the solution loses the two properties of TTI and FDT (notice that this can be seen only keeping the initial time, and thus the age of the system, fixed, while sending $N \to \infty$). The solution of the dynamical equations at low temperature was developed in [24] for the case of long range correlated noise, and then in [25] for the case of short range correlations. Roughly speaking, the main features of the solution look as follows. There exist several different ways of having the two times t and t' large while keeping a non-trivial dependance of the correlation and response. The usual asymptotic regime corresponds to having $t \to \infty$, $t' \to \infty$, with a fixed value of $\tau = t - t'$. Then the correlation and response go to their asymptotic forms $C_{as}(\tau)$ and $r_{as}(\tau)$, and represent the stationary dynamics.

In the simplest case (corresponding to 'one-step' replica symmetry breaking solutions for the statics), the interesting aging regime is unique and corresponds to the domain in which $t \to \infty$, $t' \to \infty$, with a fixed value of $\lambda = h(t')/h(t)$, where $h(t)$ is an increasing function which has not yet been determined by the theory. A possibility could be that $h(t) \propto t$, as is the case for coarsening models or in the trap model alluded to above. In this aging regime, one has $C(t,t') \sim \hat{C}(\lambda)$, and similarly for the response. This implies a response 'anomaly', which means that the system develops a long term memory. Mathematically this can be found from the following relation:

$$\lim_{t_w \to \infty} \int_0^{t_w} ds\, r(t_w, s) \neq \int_0^{\infty} d\tau\, r_{as}(\tau), \tag{13}$$

Furthermore in this aging regime the FDT is substituted by [23]:

$$Tr(t, t') = x \frac{\partial C(t, t')}{\partial t'} \tag{14}$$

where the fluctuation dissipation (FD) ratio x is smaller than one.

In more complicated cases (corresponding to 'full' replica symmetry breaking solutions for the statics), there exist several aging regimes. For instance one could take the limits $t \to \infty, t' \to \infty$, with an fixed value of $\lambda = (t - t')/t^u$. In each such regime there will be a modified FDT as in (14), with an FD ratio which depends on the regime. (For some non-understood reason the set of values of the FD ratio is related to the Parisi order parameter function in the replica solution of the statics, whenever there is a full RSB solution [24,26]).

The derivation of the aging effect from mean field dynamics, and the subsequent analytical progress, is an important breakthrough. Yet the physical mechanism underlying aging in these mean field models is not totally clear. In particular, in the simplest 'one-step' models, the dynamics is rather insensitive to temperature, and remains qualitatively the same from the dynamical transition temperature to zero temperature. This is related to the fact that aging is due to a diffusion like effect in a very high-dimensional phase space, where trapping and activated effects are absent (there is always a path to escape). In this respect, various possible scenarios have been proposed recently, including purely entropic barriers [27,28], or diffusion along basin boundaries in high-dimensional space [29]. Full RSB models are more intricate, and it would be very interesting to understand in more details what happens in these models. It thus seems necessary at this stage to develop new tools to understand the various types of aging which have been seen (a first tentative classification has been proposed in [30]). A truly challenging task, in particular, is to be able to control the finite N corrections to the mean-field models, which contain the activated effects. Precisely the same problem appears within the 'Mode-Coupling' theory of glasses [31–33].

In this respect, we want to point out the links between the mean field dynamics for a particle in a random potential and the mode coupling equations for glasses. The high temperature equations, which are the time translational equations which apply far above the freezing temperature, are the same in both cases. This might seem surprising in view of the fact that in one problem there exist a quenched disorder a priori, while in the other one there is no such disorder, it is rather 'self-induced'. In fact this difference may not be as big as it looks. Two types of recent results point into that direction. On the one hand, it has been understood that some of the hypotheses underlying the derivation of mode coupling equations just amount to assuming the existence of some hidden disorder [33] (or pseudo-random disorder [32]). On the other hand, one can find some spin systems, without any quenched disorder, which behave in all respects as bona fide spin glasses [34]. This analogy suggests to consider the low temperature mean field dynamical equations, derived for a particle in a random potential, as the generalization of the mode coupling equations well below the glass temperature. Interesting predictions on the aging behaviour of glasses can be deduced from this suggestion [33].

4. Conclusion

We have shown in this paper that the FRG and RSB techniques give compatible results when they can be compared. Both suggest quite an appealing physical picture: the phase-space of the system is, on large length scales, divided into 'cells' corresponding to favourable configurations where the potential is locally parabolic, and whose depth is exponentially distributed. This general shape can be understood through a deep analogy with Burgers' equation: elimination of the fast modes is a non linear operation which generates 'shocks' in the effective force, and 'laminar' regions between the shocks (corresponding to the cells).

Full replica symmetry breaking corresponds to the fact that these cells are themselves subdivided into smaller cells, etc. Each level of the hierarchy corresponds to a different length scale, finer details corresponding to smaller length scales.

The dynamical picture which is naively inferred from this construction landscape is that of the (multi-level) trap model [18]. The link with mean field dynamics is however not very well understood.

Another very interesting issue concerns the applicability of these ideas to spin-glasses in finite dimensions. Actually, an early version of the picture developed in the present paper was proposed within the context of one-dimensional spin-glasses in [35].

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