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On the ubiquity of spin glass concepts and methods

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We give a brief overview of some recent developments using concepts and techniques from spin glass theory in the study of other disordered systems. We discuss in particular the equilibrium correlations of a vortex lattice pinned by impurities. We mention how the replica approach, and the spontaneous breaking of replica symmetry, can be brought to bear on these problems.

There are many reasons for the long lasting interest on spin glasses [1]. On the one hand these systems display very intriguing collective behaviour, taking place on extremely long time scales. The elaboration of a full theory of spin glasses is still a very open problem, but it is already clear that the beautiful construction of a sensible mean field theory is an achievement which is important in many other fields of research where quenched disorder and conflicting interactions are at work. Neural networks, and combinatorial optimization problems, provide two examples outside of physics.

One of the main motivations for studying spin glass theory is that spin glasses should constitute a kind of archetype of systems with quenched disorder. The systems we have in mind are described by a Hamiltonian which depends on a large number (infinite in the thermodynamic limit) of quenched random variables. This creates some problems. First of all one cannot rely on symmetry arguments to find the possible phases of the system. Secondly one cannot specify in detail each sample, but one must characterize the probability distribution in the space of samples. Fortunately a large class of quantities (typically the extensive thermodynamic quantities) become sample independent in the thermodynamic limit. This has allowed the use of the replica method to compute the thermodynamic quantities in spin glasses. In spite of the apparent simplicity of this program, the real application of some of the

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powerful techniques from spin glass theory to other physical problems (with randomness and frustration) has appeared only relatively recently. One reason for that is the need for a non-perturbative method. In the replica approach one studies n identical replicas of the system, and the appearance of a spin glass phase is characterized by the spontaneous breaking of the permutation symmetry of these n replicas (for a review see [2]). Such a breaking, which is characteristic of the existence of several (meta)stable states, unrelated to each other by a symmetry, can be detected only at a non-perturbative level.

The last few years have witnessed some steps forward in this direction, using a variational approach. The difficult and fascinating problem of self-interacting random heteropolymers was studied with the replica technique and a variational method [3]. It was suggested that replica symmetry could be broken in this problem [4]. Unfortunately this is a complicated problem and one must make several approximations at various stages of the computation, so that the results on this issue of replica symmetry breaking in random heteropolymers are still not clear [5]. A much simpler problem is that of a directed polymer in an external random potential. In some sense this is a kind of mean field approximation for the problem of self-interacting heteropolymers. It is also an interesting problem for its own sake [6], which is connected to interface pinning problems and to growth phenomena [7]. A generalization of this problem, the case of directed manifolds in an external random potential, has been analyzed with the replica approach, and a variational method based on a quadratic trial Hamiltonian. It was shown that the replica symmetric solution is unstable, and a spontaneous breaking of replica symmetry à la Parisi yields a wandering critical exponent identical to that found by simple scaling arguments of the Flory type [9], but different from the one obtained through perturbation theory.

We shall illustrate the general strategy of [9], which consists in using a variational approach based on the most general quadratic Hamiltonian, but allowing for replica symmetry breaking, on the very simple toy model of one classical particle in a random potential. Denoting by ω the position of the particle, the Hamiltonian is

$$H = \tfrac{1}{2}\mu\omega^2 + V(\omega) . \tag{1}$$

V is a Brownian random potential with a Gaussian distribution characterized by its first two moments,

$$\overline{V(\omega)} = 0 , \qquad \overline{V(\omega)\, V(\omega')} = -g|\omega - \omega'| + W \equiv f(\omega - \omega') . \tag{2}$$

This model was introduced in [13] as a very simple example of an interface in a random field problem at low temperature. It is also of particular interest since it describes the asymptotic behaviour of a directed polymer in a random potential in $1+1$ dimensions [8]. It has been studied directly in [13–17]. The variational method with replicas has been studied in [10], and also recently in [11,12].

One is interested in the computation of the distribution of the particle's position. For instance one would like to compute $\overline{\langle \omega \rangle^2}$, where the overbar denotes an average over the random potential and the bracket is the thermal average. The interesting limit is the low temperature case, when g/μ^2 is large (which means that the random part of the potential dominates). A simple Imry–Ma argument [21] allows to compute the typical value ω_0 of the position of the ground state: For a displacement ω, the typical value of the potential is of order $-\sqrt{g\omega}$. So one should minimize the function $\frac{1}{2}\mu\omega^2 - \sqrt{g\omega}$, which leads to $\omega_0 \sim (g/\mu^2)^{1/3}$, so that

$$\overline{\langle \omega \rangle^2} \underset{\beta\to\infty}{\simeq} c'(g/\mu^2)^{2/3} . \tag{3}$$

It can be shown that this scaling is exact.

The problem is to try to get back this result starting from the Hamiltonian (1), and using some of the nice field theoretic methods which can then be generalized to real size finite dimensional problems. This turns out to be extremely difficult. The minimization of H leads to the equation

$$\mu\omega_0 = -V'(\omega_0) . \tag{4}$$

The natural field theoretic approach consists in trying to solve this equation by a perturbation series in V. To lowest order one gets $\omega = -V'(0)/\mu$, which leads to

$$\overline{\langle \omega \rangle^2} \underset{\beta\to\infty}{\simeq} c'(g/\mu^2) , \tag{5}$$

which is a wrong scaling. The more surprising fact is that this wrong scaling actually persists to all orders in perturbation theory. An iterative solution of the equations has also been shown to be wrong. The reason is easy to trace back: (4), which characterizes the minima of H, has many solutions in the interesting limit where g/μ^2 is large, and the usual methods of field theory do not seem to be able to handle this situation. Although the model is extremely simple, the situation is very similar to that of the random field Ising model: There, the critical exponents computed order by order in perturbation theory verify the dimensional reduction rule (they are equal to those of the pure Ising model in two less dimensions) [19,20], but this result is known to be wrong [18,22,23], because of the existence of several minima of the (free) energy [24].

The replica method starts by computing the average of the nth power of the partition function:

$$\overline{Z^n} = \int \prod_{a=1}^{n} \mathrm{d}\omega_a \exp\Big(-\tfrac{1}{2}\beta\mu \sum_{a=1}^{n} \omega_a^2 - \tfrac{1}{2} g\beta^2 \sum_{a,b} |\omega_a - \omega_b|\Big) . \tag{6}$$

Having got rid of the disorder one can go back to familiar methods. One of them is the Hartree approximation, which consists in finding the best quadratic Hamiltonian to approximate this distribution. One considers the trial Hamiltonians

$$H_\sigma = \tfrac{1}{2}\mu \sum_a \omega_a^2 - \tfrac{1}{2} \sum_{a,b} \sigma_{ab}\omega_a\omega_b , \tag{7}$$

where the σ parameters are to be determined by the stationarity condition of the variational free energy. It turns out that there are two regimes. At high temperature, or small g/μ^2, the best self-energy matrix is replica symmetric: $\sigma_{ab} = \sigma$ $(a \neq b)$ and $\sigma_{aa} = -\sum_{b(\neq a)} \sigma_{ab}$. At low temperatures or large g/μ^2, one needs to break the replica symmetry. Physically this is natural because of the existence of several solutions to eqs. (4). Technically the solution has been found using the hierarchical replica symmetry breaking scheme invented by Parisi in the framework of the mean field theory of spin glasses [2]. This solution gives back the correct scaling (3) for the position of the particle, and the prefactor which is found is off by something like 10 percent only [10]. The physical reason for this success is that a Gaussian measure in replica space, together with replica symmetry breaking, can represent a very complicated and non-Gaussian measure in physical space [9].

Recently this strategy and some variations of it have been applied to a number of interesting problems. Let us mention, apart from the ones already quoted, the random field Ising model [25] and the pinning of a vortex lattice by impurities [28]. This last problem may have particularly interesting experimental implications since decoration techniques now allow the visualisation of relatively large vortex lattices [26], and can thus provide some detailed microscopic information to be compared with theoretical predictions.

The essence of the problem is that of the equilibrium deformations of a crystal in the presence of a quenched random potential. (Another – two dimensional – experimental realization is that of magnetic bubbles in garnet films [27]). We consider a D-dimensional crystal. The equilibrium positions of the atoms at zero temperature are labelled by $\boldsymbol{x}$. Due to disorder and thermal fluctuations the atom at $\boldsymbol{x}$ is displaced to a position $\boldsymbol{r}(\boldsymbol{x}) = \boldsymbol{x} + \boldsymbol{u}(\boldsymbol{x})$. Neglecting dislocations, the Hamiltonian is the sum of an elastic term and a pinning term:

$$H = \int \mathrm{d}\boldsymbol{x} \left(\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}\boldsymbol{x}}\right)^2 + \sum_{\boldsymbol{x}} V(\boldsymbol{r}(\boldsymbol{x})) . \tag{8}$$

V is a Gaussian random potential. Its first two moments are chosen to be

$$\overline{V(\boldsymbol{r})} = 0 , \qquad \overline{V(\boldsymbol{r})\, V(\boldsymbol{r}')} = W \exp\left(-\frac{(\boldsymbol{r} - \boldsymbol{r}')^2}{2\Delta^2}\right) \tag{9}$$

and we consider here only the case where Δ is much smaller than the lattice spacing. One of the interesting questions is to understand the behaviour of the correlation function

$$C(x) = \sqrt{\overline{[\boldsymbol{u}(x) - \boldsymbol{u}(\boldsymbol{0})]^2}} \tag{10}$$

at small temperatures and large x. The same variational approach in replica space [28] has led to the following predictions: The long range positional order of the lattice is destroyed below $D = 4$. There are three regimes corresponding to different scales of $|\boldsymbol{u}(\boldsymbol{x}) - \boldsymbol{u}(\boldsymbol{0})|$, which in turn correspond to different scales of x.

At short distances $|\boldsymbol{u}(\boldsymbol{x}) - \boldsymbol{u}(\boldsymbol{0})|$ is smaller than the correlation length Δ of the potential. One can then linearize the potential and get back the random force model introduced years ago by Larkin [29]:

$$H \sim \int \mathrm{d}\boldsymbol{x} \left(\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}\boldsymbol{x}}\right)^2 - \sum_{\boldsymbol{x}} \boldsymbol{u}(\boldsymbol{x}) \cdot \boldsymbol{f}(\boldsymbol{x}) . \tag{11}$$

The scaling is then

$$C(x) \sim c' x^{(4-D)/2} . \tag{12}$$

(Notice that the discrete nature of the pinning term is important. If one would replace the discrete sum by an integral the exponent would become $(2 - D)/2$.)

The intermediate distance regime corresponds to the situation where $|\boldsymbol{u}(\boldsymbol{x}) - \boldsymbol{u}(\boldsymbol{0})|$ is larger than Δ but smaller than the lattice spacing. One cannot linearize the potential but one can consider that each atom sees a random potential which is different from the potentials seen by the other atoms:

$$H = \int \mathrm{d}\boldsymbol{x} \left(\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}\boldsymbol{x}}\right)^2 + \sum_{\boldsymbol{x}} V(\boldsymbol{x}, \boldsymbol{u}(\boldsymbol{x})) . \tag{13}$$

This Hamiltonian is just the same as the one of the random manifold problem [9] for which the replica Gaussian variational approach gives

$$C(x) \sim c' x^{(4-D)/(4+D)} . \tag{14}$$

Finally the real long distance regime described by (8) is dominated by a glassy phase where the vortices can feel they are in the same random potential. Using a regularization of the disorder, one finds the scaling

$$C(x) \sim c' x^{(4-D)/4} . \tag{15}$$

The exponents and prefactors corresponding to vortex lines have been computed. Apparently the experiments both in high-T_c and in two dimensional magnetic bubble experiments seem to probe the intermediate distance (random manifold) regime, and there is a rather good agreement with the predicted behaviour.

Obviously a lot of work still remains to be done, but I think it is interesting to see that it is now possible to build systematic microscopic approaches to these difficult problems with quenched randomness. An important open problem is to generalize this approach to the dynamical behaviour, in order to see if the situation of thermal equilibrium described here is really obtained in the experiments.

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